

# **Problem Solving Environment for Musical Instrument Design; Designing evaluation system for the acoustic characteristics of brass instruments**

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**Keywords:** CAE, Brass Instrument, Acoustics, Design, Evaluation

**Abstract** In the 21st century, the media aspect of instruments has enhanced and the needs for instruments have diversified, and they are expected to evolve into more sophisticated and high-performance instruments. For this reason, a revolutionary research and development method which is completely different from the conventional methods is demanded. This study, therefore, establishes a research and development support environment that specializes in the research and development of instruments, especially wind instruments, centered around the CAE (Computer Aided Engineering) technology and shows its effectiveness by applying it to the model simulation of instruments. The purpose of this study is to provide the environment to researchers and developers as a research and development method of instruments. In this paper, the research on the design evaluation system for the acoustic characteristics of brass instruments, especially those of trumpet.

## **1. Introduction**

For brass instruments, especially for trumpet, the good pitch characteristic (or correct interval of blown sound) is one of the basic factors as a musical instrument. But once the basic requirements are satisfied, then the tone quality and blow feeling (or easy sounding, adequate blow resistance, etc.) become important. Instruments are evaluated in terms of sensibility as a synergistic effect of the playing technique of a player and the acoustic characteristics of a sound tube [1].

The studies of methods to analyze and evaluate the acoustic characteristics of brass instruments include artificial blowing models and their analyses [2, 3] and computer simulations of artificial blowing models [4, 5 and 6]. All of the studies remain within the scope of the fact analyses and descriptions of phenomena of actual instruments and have not reached to the level of proposals which improve the acoustic characteristics and/or design methods of brass instruments.

It is said that the mouthpiece has a substantial influence on the blow feeling as well as tone quality. There are several foregoing studies in which the relationship between the cup depth of a trumpet's mouthpiece and its tone quality was analyzed [8] and the shape of the mouthpiece of a horn, its acoustic characteristics and blow feeling was analyzed [9]. In all of these studies, actual instruments and their mouthpieces were used and the difference of characteristics of tone qualities was examined by measuring and analyzing them. And the correlations between the peak level and Q value of input impedance characteristics to tone quality were shown and the usefulness of

simulation method to reveal the acoustic characteristics of the wind-instrument system was stated.

The purpose of this study is to predict and evaluate the acoustic characteristics of a virtual tube body during the design phase by computer simulation. Now, we assume that “A good trumpet has good tone quality, sound volume (dynamic range), and sound production performance (response) and can be blown with rich tone quality and good blow feeling.” Among these factors, this study focuses on the acoustic characteristics of a sound tube and shows its analysis method and the examples of analysis. Periodic pulses from a mouthpiece of a wind-instrument system is assumed and the effects of the mouthpiece shape on the sound volume, sound quality and volume of radiated sound and blow feeling are described. In Chapter 2, the analysis theory of transfer characteristic of sound pressure within a tube body is presented with a case example of popping sound analysis. In Chapter 3, the application fields of this theory and their impacts are presented. First, regarding the steady sound pitch of brass instruments, the tube body of a trumpet is modeled by combining conical horn elements and then the resonance frequency of the modeled tube body is calculated using the cavity resonance method assuming the mouthpiece end as closed. Next, the effects of the mouthpiece shape, which is said to have a large influence on tone quality and blow feeling, are studied to evaluate tone quality and blow feeling. Finally, the engineering model during the design and manufacture process of a brass instrument is proposed. In Chapter 4, conclusion is stated stressing the importance of the design and evaluation system in the future musical instrument design.

## 2. Sound Tube Theory of Brass Instruments

### 2.1 Sound Pressure, Sound Particle Velocity and Acoustic Impedance within Sound Tube

In a tube body of a brass instrument, the steady sound wave in its straight tube section without nonlinear phenomena can be considered as a plane wave along the axis of the tube body. Its bent tube part is equivalent to a straight tube which has the same length of axis if the wavelength of its sound wave is substantially longer than its curvature radius and its steady wave can be considered as a plane wave. Since the internal shape of a brass instrument can be approximated by a serially connected truncated conical elements [3], the acoustic impedance etc. within the conical horn is derived at first. Note that a cylindrical tube element is treated as a special case of a truncated conical element.

It is known that the velocity potential  $\phi$  of a sound wave in a straight tube whose sectional area is defined by  $S(x)$  is governed by the Webster wave equation below [11],

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial \phi}{\partial x} \frac{\partial (\log S)}{\partial x} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2}$$

where  $c$  is the sonic speed. Especially, in the case of a conical horn (conical tube) or  $S = \pi(Ax)^2$  where  $S$  is the section area at central axis coordinate  $x$  and  $A$  is a positive real number, the equation can be expressed as follows.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{2}{x} \frac{\partial \phi}{\partial x} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2}$$

Then, the solution of velocity potential when the sound wave vibrates in a sine-wave with an angular frequency of  $\omega$  can be expressed as follows,

$$\phi = \frac{1}{x} \left( \Phi_+ e^{-jkx} + \Phi_- e^{jkx} \right) \cdot e^{j\omega t}$$

where the wave number  $k$  is  $\omega/c$ .

Note that  $\Phi_+$  and  $\Phi_-$  are the complex amplitudes of the velocity potentials of the sound waves which propagate to the positive and negative directions of  $x$ , respectively, and can be determined by the

boundary condition. They are the constants independent of  $x$ . Here, given  $-\Phi_+/\Phi_- = e^{-2\varphi}$ , the equation can be arranged as follows.

$$\phi = \frac{\Phi_-}{x} \left( e^{\varphi + jkx} - e^{-(\varphi + jkx)} \right) \cdot e^{-\varphi + j\omega t}$$

Here,  $\varphi$  is a complex number and its real parts is the square root of the absolute value of  $\Phi_+/\Phi_-$  and its imaginary part is the half value of the phase difference of the two. By using  $\varphi$ , the traveling wave and reflection wave can be expressed so that their spatial phases are opposing each other with reference to the intermediate phase between  $\Phi_+$  and  $\Phi_-$ . Assume that  $\varphi$  is expressed as follows.

$$\varphi = \pi\alpha_0 - j\pi\beta_0$$

The  $-2\pi\beta_0$  indicates the phase angle between the traveling and reflection waves at  $x = 0$ .

Using the functions:  $\sinh(x) = (e^x - e^{-x})/2$ ,  $\cosh(x) = (e^x + e^{-x})/2$ ,  $\tanh(x) = \sinh(x)/\cosh(x)$  and  $\coth(x) = \cosh(x)/\sinh(x)$ , the equation can be expressed as follows.

$$\phi = \frac{\Phi_-}{x} \sinh(\varphi + jkx) \cdot e^{-\varphi + j\omega t}$$

Therefore, the sound pressure  $p$  and the sound particle velocity  $\dot{\xi}$  are expressed by the following equations.

$$p = \rho \frac{\partial \phi}{\partial t} = 2j\omega\rho \frac{\Phi_-}{x} \sinh(\varphi + jkx) \cdot e^{-\varphi + j\omega t}$$

$$\dot{\xi} = -\frac{\partial \phi}{\partial x} = 2\Phi_- \left( \frac{1}{x^2} \sinh(\varphi + jkx) - \frac{jk}{x} \cosh(\varphi + jkx) \right) \cdot e^{-\varphi + j\omega t} \quad (2.1)$$

Here,  $\xi$  is displacement,  $\rho$  is air density, and  $k = 2\pi f/c$ , where  $f$  is frequency.

Hence the acoustic impedance (specific acoustic impedance)  $Z$  is described as follows.

$$Z = \frac{p}{\dot{\xi}} = -\rho c \frac{1}{j/kx + \coth(\varphi + jkx)} \quad (2.2)$$

Here, let  $Z$  be as follows.

$$Z = \rho c \tanh \theta$$

$\rho c$  is the characteristic impedance of sound wave in the air. Let's call  $\theta$  as complex phase.

Now, given that the damping due to friction resistance etc. cannot be ignored, Equation (2.2) can be modified as follows by considering the damping term  $\alpha(x)$ .

$$Z = -\rho c \frac{1}{j/kx + (1 + \alpha'/jk) \coth(\varphi + jkx + \alpha(x))} \quad (2.3)$$

$$\alpha' = \partial \alpha(x) / \partial x$$

The following equation is the experimental equation recommended by Beranek and  $r$  is a coefficient determined by the characteristics of a sound tube.

$$\alpha(x) = r \times \sqrt{k} \times x / D$$

$D = (D1 + D2) / 2$ ; where  $x$  is the axial length of element, and  $D1$  and  $D2$  are the diameters of an element at both ends.

## 2.2 Sound Pressure and Transfer Function

Next, consider a steady state which is obtained over time based on the traveling wave from a

sound source placed in the direction of 0 in Fig.1 and repeatedly reciprocate in the tube. Let a sound wave with a frequency and a known sound pressure at one end I of a sound tube element be  $p(x_1)$  and the sound pressure at the opposite end II  $p(x_2)$ .

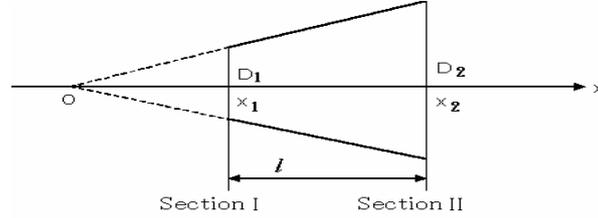


Fig.1 Sound Tube Element

From Equation (2.1), the sound pressure of an arbitrary section can be expressed as follows by considering the damping term  $\alpha(x)$ .

$$p(x) = j\omega\rho \frac{2\Phi}{x} \sinh(\varphi + jkx + \alpha(x - x_1)) \cdot e^{-\varphi + j\omega t}$$

Hence the sound pressure  $p(x_2)$  is expressed as follows.

$$p(x_2) = p(x_1) \frac{x_1 \sinh(\varphi + jkx_2 + \alpha(l))}{x_2 \sinh(\varphi + jkx_1)} \quad (2.4)$$

Here, let the complex phase  $\theta_1$  at the section I be known and  $Z_1 = \rho c \tanh \theta_1$ , then  $\varphi$  can be expressed as follows from Equation (2.3).

$$\varphi = -\coth^{-1} \left( \frac{1}{(1 + \alpha' / jk)} (\coth \theta_1 + j / kx_1) \right) - jkx_1 - \alpha(x - x_1)$$

Since the value of  $\alpha$  is zero and  $\alpha' \ll 1$  at the section I, the equation above can be approximated as follows using the relation  $\varphi_1 = \varphi + jkx_1$ .

$$\varphi_1 = -\coth^{-1} (\coth \theta_1 + j / kx_1) \quad (2.5)$$

Here,  $\varphi_1$  indicates the phase at the section I of the sound tube element. Since  $x_2 = x_1 + l$ , Equation (2.4) can be expressed by the following equation.

$$p(x_2) = p(x_1) \frac{x_1 \sinh(\varphi_1 + jkl + \alpha(l))}{x_2 \sinh(\varphi_1)} \quad (2.6)$$

This equation means that the phase at the section II goes ahead of  $\varphi_1$  at the section I by  $(\alpha(l) + jkl)$ . Furthermore, from the relationship  $Z_2 = \rho c \tanh \theta_2$  and Equation (2.3),  $\theta_2$  can be expressed as follows.

$$\theta_2 = \tanh^{-1} \frac{-1}{(1 + \alpha' / jk) \coth(\varphi_1 + jkl + \alpha(l)) + j / kx_2} \quad (2.7)$$

When the section II and the next element's section I are discontinuous, the continuity of sound pressures and the relationship of constant volume velocity at the discontinuous boundary are maintained. In such case, the acoustic impedance is converted according to the reverse ratio of the sectional area change to determine the complex phase  $\theta_1$  at the next element's section I.

The relationships of  $x_1$ ,  $x_2$ ,  $D_1$  and  $D_2$  are as follows.

When  $D_1 < D_2$ : the apex of the cone is at the left side of the section I and relations can be expressed

as follows.

$$x_1 = \frac{(D_1/D_2) \cdot l}{(1 - D_1/D_2)} \quad x_2 = \frac{l}{(1 - D_1/D_2)}$$

The relationship of  $x_1/x_2 = D_1/D_2$  holds and this indicates the degree of taper of the element.

When  $D_1 > D_2$ : the apex of the cone is at the right side of the section II and the equation is the same as in the case of  $D_1 < D_2$ , but the values of  $x_1$  and  $x_2$  are negative.

When  $D_1 = D_2$ : the element becomes a cylindrical tube and is not a truncated cone anymore. For example, in the case of  $D_1 < D_2$ , it can be considered as a limit state where  $D_1$  and  $D_2$  have very close values,  $x_1$  and  $x_2$  are large enough while maintaining the relationship  $x_1 - x_2 = l$ . Therefore, by taking the limits of  $x_1 \rightarrow \infty$  and  $x_2 \rightarrow \infty$  in Equation (2.5) and (2.7), the following relations hold.

$$\begin{aligned} \varphi_1 &= -\theta_1 \\ Z_2 &\cong -\rho c \tanh(\varphi_1 + jkl + \alpha(l)) \\ \theta_2 &\cong -(\varphi_1 + jkl + \alpha(l)) \end{aligned} \quad (2.8)$$

Let the transfer function of sound pressure of the i-th sound tube element be  $GM_i(\omega)$ , it can be expressed as follows using Equation (2.6).

$$GM_i(\omega) = \frac{p(x_2)}{p(x_1)} = \frac{x_1}{x_2} \frac{\sinh(\varphi_1 + jkl + \alpha(l))}{\sinh(\varphi_1)}$$

$$p(x_2) = p(x_1) \times GM_i(\omega) \quad (2.9)$$

If the sound tube consists of n-elements, the transfer function of the sound pressure of the whole sound tube elements is expressed as follows.

$$G(\omega) = \prod_1^n GM_i(\omega) \quad (2.10)$$

Assuming that the bell end of the whole sound tube is open and the mouthpiece end is closed, we set boundary conditions of the open and closed ends. If the element of the closed end is approximated by an extremely short cylindrical tube and the closed end is rigid, the acoustic impedance is infinite and the value of the closed end's complex phase  $\theta$  is  $(0, -j\pi/2)$ .

The boundary condition of the open end during resonance was expressed as follows in Equation (24.18) of Reference [7] by assuming a piston motion within an infinite baffle plane, and let the radiation impedance be  $Z_{rad}$ .

$$Z_{rad} = \rho c \left[ 1 - \frac{2J_1(w)}{w} \right] - j\rho c M(w)$$

Here,  $w = k \cdot D$ :  $D$  is the diameter at the open end, and  $J_1(w)$  is the first order Bessel function.

$$M(w) = \frac{4}{\pi} \int_0^{\pi/2} \sin(w \cos \alpha) \cdot \sin^2 \alpha d\alpha$$

Let the acoustic impedance with which the inside of the tube is seen from the open end be  $Z_{out}'$  and let the acoustic impedance of the open end be  $Z_{out}$ , then the relationship between them can be expressed as  $Z_{out} = Z_{out}' - Z_{rad}$ . Since  $Z_{out} = 0$  during resonance, its complex phase  $\theta_{out}$  becomes  $(0, -j0)$ .

### 2.3 Modeling Accuracy Check through Analysis of Popping Sound and Calculation of Sound Pressure Transfer Characteristics

The tube body of a trumpet is modeled by combining cylindrical tube elements and truncated conical elements. The bent tube part is modeled by a straight tube with an equivalent length along its center line. The cylindrical tube part is modeled by one acoustic tube model regarding its length. As for the parts where the rates of sectional area change are big such as the open flare part and mouthpiece cup, the elements are finely defined with a length of 1 to 2 mm to maintain the NC machining accuracy of jigs and tools necessary for manufacturing. About 160 elements are used for modeling the whole tube except for the tuning slide tube part. An example of the modeling of a piston valve trumpet is shown in Fig. 2.

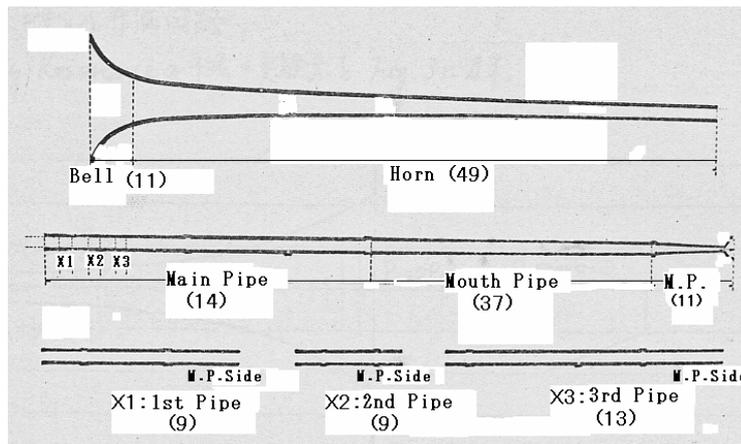


Fig.2 Modeling of Tube Body Shape

In the figure above X1, X2, and X3 are the first, second, and third tuning slide tubes selected by the piston fingering. M.P. indicates the mouthpiece, and the numbers in parentheses are the numbers of elements in each part

For this C-major trumpet “C02,” three types of mouthpiece are used; “3C”: Standard shape, “1A”: Deeper cup, and “9E”: Shallower cup (Fig. 3). The throat diameters are 3.78 mm for “3C” and “9E”, and 4.00 mm for “1A.” The throat lengths and the back bore shapes vary according to types, but the total length of the mouthpiece is common and is 87.5 mm.

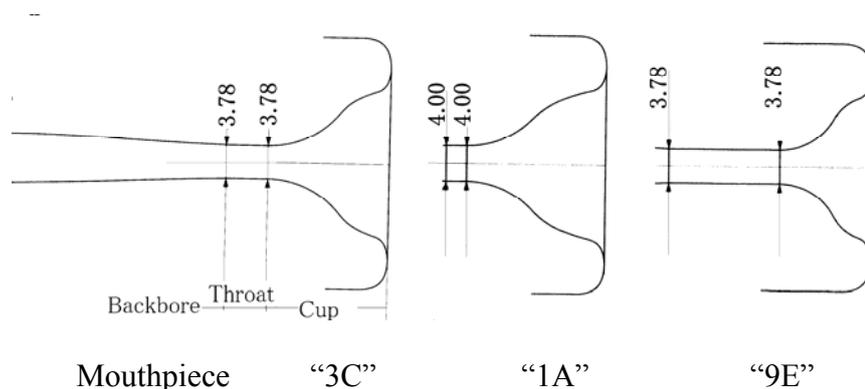


Fig.3 Shapes of Mouthpieces Used in Experiment

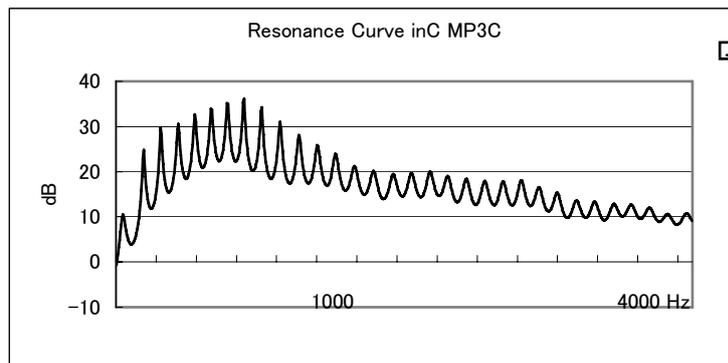
The volumes of the cup of mouthpiece and the whole mouthpiece are as follows (Table 1.).

Table 1. The volumes of the cup of mouthpiece and the whole mouthpiece(Unit:  $cm^3$ )

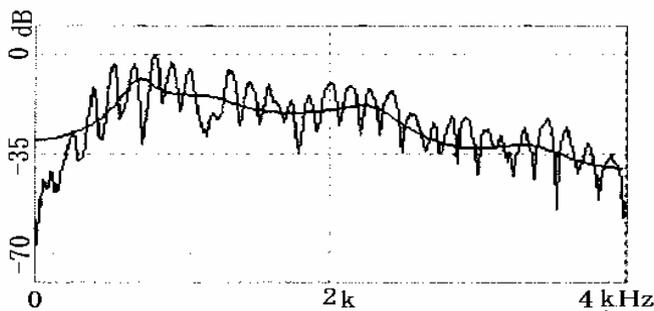
M.P.	“3 C”	“1 A”	“9E”
Cup	1.36	1.74	0.96
M.P.Total	3.48	4.28	3.06

When panting the mouthpiece end with a palm, the “initial percussive” and the “later resonance in tube” occur. The subject of this study is the latter. Here, the modeling accuracy is examined from both the actual measurement and the calculation of sound pressure transfer characteristics obtained by analyzing popping sound. In Reference [10], it is described that the effective elongation ( $L_e$ ) of the mouthpiece is determined by the volume of the mouthpiece in the low-frequency range and is determined by the popping sound of the mouthpiece itself at the popping frequency (fundamental frequency of the response when an impulse input is applied at the cup end of the mouthpiece unit).

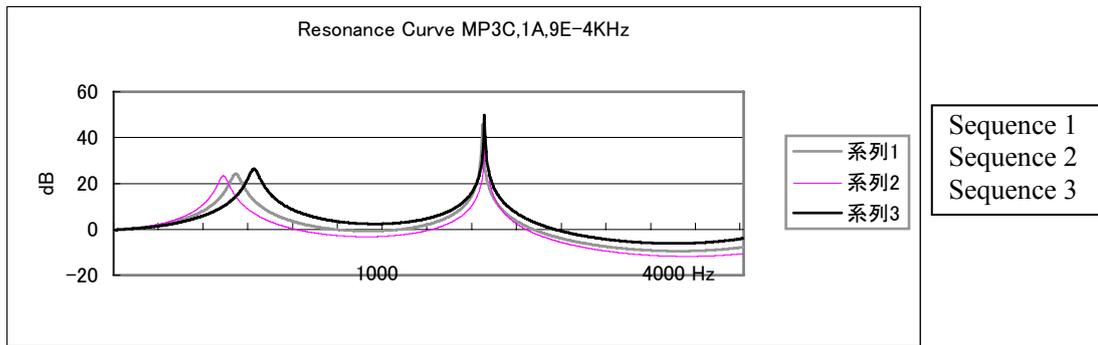
Fig. 4(a,b,c,d) shows the calculated and measured example of the sound pressure using Equation (2.10) and the FFT analysis example of the popping sound for the modeled tube body “C02” when no piston is pressed (Open fingering).



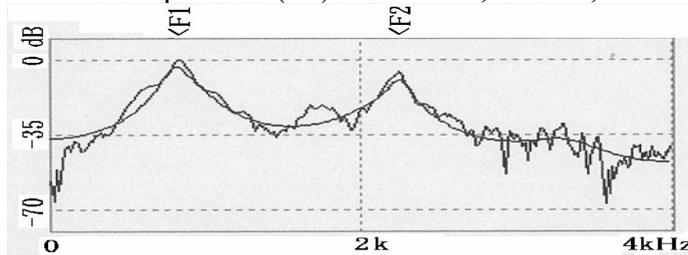
a) Sound Pressure Transfer Characteristic of Whole Tube Body Including M.P.; C tube with (Mouthpiece “3C”)



b) Actual Measurement Example of Popping Sound of Whole Tube Body Including M.P.; C tube (with Mouthpiece “3C”) The thick solid line indicates an envelope curve.



- c) Calculated Values of Sound Pressure Transfer Characteristics of M.P Unit  
 The sequences 1, 2, and 3 are correspond to the mouthpieces “1A,” “3C,” and “9E,” respectively.  
 The formant frequencies (F1) are 741 Hz, 820 Hz, and 933 Hz, respectively.



- d) Actual Measurement Example of Popping Sound of M.P. Unit (with Mouthpiece “1A”)  
 The thick solid line indicates an envelope curve. F1 and F2 indicate the formant frequencies. F1 = 781 Hz, F2 = 2,265 Hz

Fig. 4 Examples of Calculated Value of Sound Pressure Transfer Characteristic and Actual Measurement of Popping Sound for Whole tube body including M.P. and M.P. Unit

The calculated value was obtained by setting  $r = 1.0 \times 10^{-4}$  in Equation (2.3), by inputting sine waves with a constant sound pressure from the mouthpiece end, by applying Equations (2.3), (2.5), and (2.6) to each element sequentially, and finally by calculating the frequency characteristic of the radiated sound pressure (dB) at the open end using Equation (2.10). The closed end of the mouthpiece is assumed as closed by an elastic thin plate and its complex phase is assumed as  $\theta_{in} = (0, j\pi/2)$ . The boundary condition at the open end is assumed as a piston motion within an infinite baffle plane (Radiation impedance:  $Z_{rad}$ ). At resonance, the complex phase of the impedance  $Z_{out}$  which is compensated at the open end by  $Z_{rad}$  is assumed as  $\theta_{out} = (0, -j0)$ .

The frequency which shows the maximum value of the sound pressure is the resonance frequency of the steady wave of a sound tube, and the resonance points are located at the frequencies of integral multiple of the fundamental frequency on the frequency axis. In actual blowing, however, the pitch does not always agree with this resonance frequency.

It can be said that the actual measurement of the popping sound shown in Fig. 4 is fairly corresponding to the calculated value of the sound pressure transfer characteristic using Equation (2.10). But the measured popping frequency become higher than the calculation, because the effective length of the mouthpiece varies as the cup volume decreases due to intrusion of the effective closed end to the cup when the mouthpiece is smacked with a palm [11]. In the case of the value for the mouthpiece unit, the result of “3C” is F1 = 843 Hz and this value corresponds to the calculated value of the primary resonance frequency when the cup end is shortened by 1.3 mm.

The models in which the bore diameter of the bell part was changed by  $\pm 1\%$  were also put to calculation, and the changes of the characteristic values were very small.

According to the results above, it can be said that the modeling accuracy and the analysis method are appropriate. Players and manufacturer may sometimes check the resonance of an instrument by the popping sound. Instead of the observation of the popping sound, the acoustic characteristics can be strictly estimated and evaluated from the transfer characteristic of the sound pressure.

### **3. Application of the Theory and Impacts**

The first application of this theory is the pitch calculation of the steady sound of brass instruments. The resonance frequency as the characteristic value of a model tube body is calculated by modeling the tube body as a combination of conical horn elements, and by assuming the mouthpiece end is closed, and by using the cavity resonance method. The second application is the calculation of the effective input impedance of the mouthpiece and its phase angle by setting a blowing pattern as the boundary condition at the mouthpiece end, to get the pitch at actual blowing, and qualitatively examine the effect of blowing methods. For this purpose, a hypothetical additional length at the mouthpiece representing individual blowing methods must be postulated. By incorporating this method to the pitch design system, one can verify the effectiveness of the system to the design of an actual trumpet at blowing stage.

The third application is the analysis of the effect of the mouthpiece shape on tone quality and blow feeling. Different types of mouthpieces are modeled and the relationship between the tone qualities and blow feelings due to the difference of their characteristics is examined, and then its physical meaning can be investigated. The transfer characteristic of the whole mouthpiece including the cup, throat, and back bore makes a large contribution to the formation of tone quality. One of the factors which determine the easiness of blowing is the acoustic impedance at the mouthpiece end, and the factor which determines blow resistance feeling is the sound particle velocity at the throat. By incorporating this method to the tone quality and blow feeling design system, one can get useful information for the design of a high-quality trumpet.

One of the authors has proposed the engineering model in the design and manufacturing process of brass instruments [12]. Design processes of an actual trumpet are divided into two categories; original design and copy design. The latter process is what is called as reverse engineering. To improve the development efficiency, it is mandatory to set the target accurately, and take advantage of the analysis evaluation tools, and to use 3D shape design tool as a premise. Especially, the evaluations related to tone quality and performance easiness are important and more scientific analysis methods are desired. By accumulating information through the use of such tools, the promotion of development efficiency and the inheritance and advance of technology can be achieved. In the age when the modality of manufacturing is questioned, the development of high-value instruments meeting the diverse needs which are different from the conventional mass production instruments is also be desired in the field of instruments manufacturing.

### **4. Conclusion**

The characteristics of an instrument as a sound tube can be understood as the transfer function  $G(\omega)$  of the sound pressure in the tube body including the mouthpiece shape. Although the transfer function  $G(\omega)$  of the tube body contributes to the formation of tone quality and then has an impact on its blow feeling, it can be said that the effect of the shape of the mouthpiece, which is closer to a player, has larger impact on blow feeling.

The feature of the transfer characteristic of the mouthpiece corresponds well with the difference of tone quality and blow feeling due to the mouthpiece. By using these characteristics as evaluation functions for actual design, a design theory which associates the shape design and the blow feeling including tone quality would be established. The standard design method of a mouthpiece will be as follows. Starting from the reference tube body and a mouthpiece, the cup shape is determined by using blow feeling and tone quality as evaluation functions, then the throat shape is determined. Next, the bore shape and length of the back bore is determined to adjust the pitch characteristic (pitch and balance), then the transfer characteristic would be evaluated again.

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